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A modified density approach for topology optimization in magnetic fields

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Abstract

Topology optimization has been widely used for the optimal structure design not only in elastic fields but also in electric/magnetic fields. The homogenization design method and the density approach are the representative methods in topology optimization. The purpose of this work is to suggest a modified density approach for the topology optimization in magnetic fields considering the characteristics in magnetic fields. In this work, a hole is assumed in an element based on the concept of the homogenization design method, and the density and the magnetic permeability of the element are obtained according to the size of the hole at each element while only one design variable, the element density, has been considered in the ordinary density method. It is also addressed that the appropriate value of the penalization parameter is quite different in magnetic optimization problems comparing to the value in elastic optimization problems. Numerical results according to the element density and the design domain are also discussed.

Keywords: Topology optimization; Homogenization design method; Density approach; Magnetic fields; Penalization parameter

1. Introduction

Since the work by Schmit (1960), the constrained optimization combined with the finite element method (FEM) has been widely used in structural optimization, more particularly to reduce the weight of the structure still satisfying the physical requirements. In the structural optimization in continuously variable design fields, the requirement of the relaxation of the design domain was suggested by Cheng and Olhoff (1981). To satisfy the requirement, Bendsøe and Kikuchi (1988) developed a new method entitled as the homogenization design method (HDM) using the homogenization theory. Since then, HDM has become the most common approach to obtain the optimal topology in elastic fields such as static (Suzuki and Kikuchi, 1991), dynamic (Díaz and Kikuchi, 1992; Ma et al., 1993) and buckling problems (Neves et al., 1995, 2002). In HDM, the design domain is assumed to have infinite number of micro-structures and the material properties are homogenized. The density approach has become popular because of its conceptual

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simplicity. Similar to HDM, it divides the design domain with finite elements. However, only the densities of each of the elements are selected as design variables and the intermediate values of the density are penalized (Mlejnek and Schirrmacher, 1993).

Both HDM and the density approach are based on two numerical methods: FEM and the optimization algorithm. The former one is used to analyze a field discretized by finite elements and the latter one is to obtain the optimal values of the design variables considering the objective and the constraints. It is also possible to obtain the optimal structure under the effect of magnetic fields (Marrocco and Pironneau, 1978) as well as to analyze the magnetic field by FEM. In the structural optimization in magnetic fields, the density approach has been widely used. Dick and Lowther (1996, 1997) used the density method to optimize the magnetic devices and entitled the method as the optimal material distribution method (OMD). This method has been also used to obtain the optimal topology of a structure in electric fields (Byun et al., 1999, 2000, 2002). However, the results are dependent on the discretization density of the design domain. Furthermore, some of the final topologies show vague boundary shapes especially in case of topology optimization in magnetic fields. Recently, HDM has been applied for topology optimization in magnetic fields to maximize the magnetic energy. Yoo and Kikuchi (2000) applied HDM to obtain the optimal topology of a magnetic device in linear cases. The work by Yoo et al. (2000) expand the application of HDM into non-linear cases to consider the saturation effect of the magnetic devices. HDM is also used to minimize the vibration caused by forces induced by magnetic fields (Yoo, 2002). In both methods, the material property such as magnetic permeability is used to determine the energy in the design domain as well as optimal topology in magnetic fields while the elastic modulus is used in elastic fields. Although the results by HDM show clear final shapes, the application of HDM requires complex pre-processing to obtain the homogenized permeability value.

In HDM, the design domain is composed of finite number of finite elements and each of the elements is assumed to have infinite number of micro-structures. A rectangular hole is located in the micro-cell and the parameters defining the size and the rotational angle of the hole are design variables. Therefore, the number of design variable is theoretically infinite. Due to this basic concept, topology optimization by HDM involves working with orthotropic or anisotropic materials. If the density approach is used, the results are dependent on mesh size since the design domain is divided into finite number of finite element. Kim and Kim (2000) suggested the transformation of direct density variables into new variables by wavelet transform to resolve the deficits of the traditional density approaches such as mesh dependencies and local minima. However, in the ordinary density method, each element in the design domain is basically considered as isotropic since the only design variable of an element is the density and no additional refinement of the design domain is required. If the magnetic flux in magnetic fields or wave-guide problems in electromagnetic fields are considered, the directivity of the flux or the wave is quite important. As mentioned above, we have little flexibility to control such directional properties if the traditional density approach is used for the optimization.

In case of topology optimization in elastic fields using the density approach, the value of the penalization parameter is generally taken as 2–4 to push the intermediate densities to either lower or upper bounds (Bendsøe and Sigmund, 1999; Haber et al., 1996). Kim and Yoon (2000) performed particular work to get a better topology by the density method using wavelet transform. Similar value of the penalization parameter is used in topology optimization in magnetic fields (Byun et al., 1999). The magnetic flux and the elastic strain are used to compute the magnetic energy and elastic energy, respectively. Furthermore, the objective function used in topology optimization is deeply related with the energy in the design domain. The elastic strain is determined by the divergence of the deformation while the curl of magnetic vector potential determines the magnetic flux and both the deformation and the magnetic vector potential are initial results obtained by the finite element analysis. Considering the differences and the similarities mentioned above, it is necessary to check whether it is right or not to assign the similar value of penalization parameter in magnetic field optimization as in elastic field optimization.

In case of magnetic field problems, it is necessary to consider the physical phenomena that have different properties according to the directions. Up to the present, no work for topology optimization in magnetic fields using the density method could reflect orthotropic or anisotropic material properties. This work suggests the modified density method to reflect the directional properties in magnetic field optimization. The concept of the new approach is assuming a hole in an each finite element of the design domain and making the parameters that determine the size of the hole as the design variable utilizing the HDM concept. Therefore, we can deal with orthotropic or anisotropic material by applying the modified method. If we desire to obtain the black-and-white results for the initial figure of a structure, the modified density approach can lead to designs with little gray portion as HDM does.

In elastic problems, usually 2–4 is taken as the value of the penalization parameter and similar value is used in magnetic field problems without further consideration. In this work, the selection of an appropriate penalization parameter for topology optimization in magnetic fields is studied. The topology optimization results by HDM and the modified density methods with different penalization values are compared. The final result shows quite different tendency in magnetic field problems. The change of the mesh discretization density as well as the variation of the design domain is studied to verify the effects into the final results.

2. Topology optimization in magnetic fields

Topology optimization can be simply defined as a design process to determine the optimal material distribution in a give design domain. Different to shape optimization (Haug et al., 1986), the design domain is generally fixed during the optimization process. Fig. 1 illustrates the concept for topology optimization in magnetic fields by HDM and the modified density approach in two-dimensional cases. The external current density $\boldsymbol{J}$ is applied along the boundary region Γ_J and Γ_d is the fixed region with Dirchlet or Neumann boundary conditions. $\boldsymbol{x}(x_1, x_2)$ represents the coordinate system in macro-scale while $\boldsymbol{y}(y_1, y_2)$ represents the coordinate system in micro-scale. The design domain Ω is discretized by finite number of finite elements. If HDM is used, each of the finite elements is assumed to be composed of infinite number of unit-cells as shown in the figure. On the contrary, if the ordinary or modified density approach is applied, we do not need to consider the micro-structure anymore. The density of each finite element is defined as a unique design variable and computed according to the size of the hole located in the element.

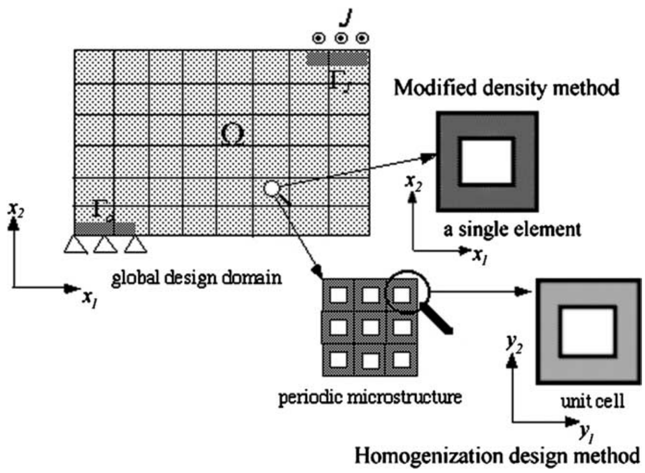

Fig. 1. Concept of topology optimization by HDM and the modified density method.

In HDM, the unit-cell has a rectangular hole as shown in Fig. 1 and the size and rotational angle of the hole are design variables. Since each finite element in the design domain is assumed to have countless unit-cells, the material properties in the element are homogenized. Suzuki and Kikuchi (1991) derived the homogenized Young's modulus E^{H}_{ijkl} and the homogenized density ρ^{H} as follows:

$$E^{\mathrm{H}}_{ijkl} = \frac{1}{|Y|}\int_Y \left(E^0_{ijkl} - E^0_{ijkl}\frac{\partial \chi^{kl}_p}{\partial y_q} \right) \mathrm{d}Y \tag{1}$$

$$\rho^{\mathrm{H}} = \frac{1}{|Y|}\int_Y \rho_0 \,\mathrm{d}Y \tag{2}$$

where Y and $|Y|$ stand for unit-cell and the volume of the unit-cell, respectively. E^0_{ijkl} is the real Young's modulus of the original material while ρ_0 is the real material density. χ is the characteristic function representing the deformation of a unit-cell. In case of an elastic problem, the strain energy of a design domain can be expressed as

$$U_{\mathrm{elastic}} = \sum_{e=1}^{N_e} \frac{1}{2}\int_{\Omega_e} \varepsilon_{ij} E^{\mathrm{H}}_{ijkl} \varepsilon_{kl} \,\mathrm{d}\Omega \tag{3}$$

where ε_{ij} and ε_{kl} are strain tensors. The number of elements in the design domain is denoted by N_e and Ω_e represents the region occupied by a finite element.

For topology optimization in magnetic fields, the magnetic permeability is homogenized and used to compute the magnetic energy. Setting μ_{ij} as the original magnetic permeability, the homogenized permeability can be expressed as follows (Yoo and Kikuchi, 2000):

$$\mu^{\mathrm{H}} = \int_Y \mu_{ij} \,\mathrm{d}Y + \int_Y \mu_{ij}\frac{\partial \gamma_i}{\partial y_j} \,\mathrm{d}Y \tag{4}$$

In this case, the characteristic function γ represents the magnetic field strength of a unit-cell. The magnetic energy of a design domain can be written as

$$U_{\mathrm{magnetic}} = \sum_{e=1}^{N_e} \frac{1}{2}\int_{\Omega_e} \frac{1}{\mu^{\mathrm{H}}_{ij}} B_i B_j \,\mathrm{d}\Omega \tag{5}$$

where B_i and B_j are magnetic flux densities.

If the density method is used for the optimization, the material properties are defined for each of the finite elements composing the macro-size design domain. Assuming the real material density as ρ_0, the intermediate Young's modulus is usually penalized as (see, Yang, 1997);

$$E = E_0 \left(\frac{\rho_i}{\rho_0} \right)^p \tag{6}$$

where ρ_i represents the intermediate material density. E and E_0 are the intermediate and original Young's moduli, respectively. The penalization parameter is usually required to be equal or greater than 3. Bendsøe and Sigmund (1999) determine the value according to the Poisson's ratio of the material considering the Hashin–Shtrikman bounds for two-phase materials. If the material property is orthotropic, the Young's modulus is characterized for three-dimensional cases as follows:

$$E = \begin{bmatrix} E_{1111} & E_{1122} & E_{1133} & 0 & 0 & 0 \\ & E_{2222} & E_{2233} & 0 & 0 & 0 \\ & & E_{3333} & 0 & 0 & 0 \\ & & & E_{2323} & 0 & 0 \\ & & & & E_{3131} & 0 \\ \mathrm{SYM} & & & & & E_{1212} \end{bmatrix} \tag{7}$$

Referring to Byun et al. (1999), the intermediate magnetic permeability in magnetic fields may be expressed as

$$\mu = \mu_0 \left[1 + (\mu_r - 1) \left(\frac{\rho_i}{\rho_0} \right)^p \right] \tag{8}$$

where μ_0 and μ_r represent the magnetic permeability in free space and relative magnetic permeability, respectively. The penalization parameter has been valued as 2–4 referring the optimization results in elastic fields.

3. The modified density approach

In HDM, a rectangular hole is located at each of the micro-cells and the rotational angles and the size of the hole are design variables. In the new approach, a hole is assumed at each of the finite elements composing the macro-design domain.

Fig. 2 shows the schematic figure of a finite element in the design domain. In three-dimensional cases, the design domain is divided by eight-node hexahedral elements and the size is normalized as shown in the figure. If the design variables are d_1, d_2, d_3 which describe the hole-size, the element density can be computed as

$$\rho = \rho_0 (1 - d_1 d_2 d_3) \tag{9}$$

where ρ_0 represents the real material density. In the black-and-white representation for the optimal topology, the element is colored as black if the hole-size becomes 0. By normalizing the density value and using Eqs. (8) and (9), the magnetic permeability may be expressed as follows:

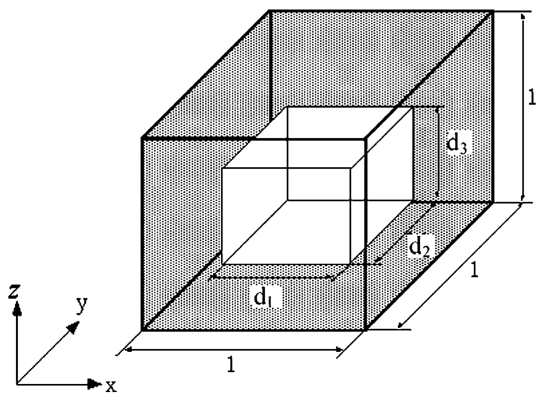

Fig. 2. Figure of an element for modified density approach.

$$\mu = \mu_0[1 + (\mu_r - 1)\rho^p] = \mu_0[1 + (\mu_r - 1)(1 - d_1 d_2 d_3)^p] \quad (10)$$

As mentioned in the previous section, the penalization parameter p is usually selected as 2–4. Fig. 3 shows the variation of the normalized Young's modulus by changing the element density. For the results by density method penalization values are selected as 1, 2 and 3 and the result of E_{1111} by HDM is also displayed. As can be confirmed from the figure, the results by HDM and the density method are similar for large penalization values such as 2 or 3.

Fig. 4 shows the variation of the magnetic permeability by HDM and the modified density method according to the element density and the penalization parameter is defined as 0.5, 1, 2 and 3. The permeability values are obtained assuming an isotropic material both by HDM and by the modified density approach. Different to the result in Young's modulus case, the result by HDM is mostly well matched with the results by the modified density approach with small penalization parameters such as 0.5 or 1.

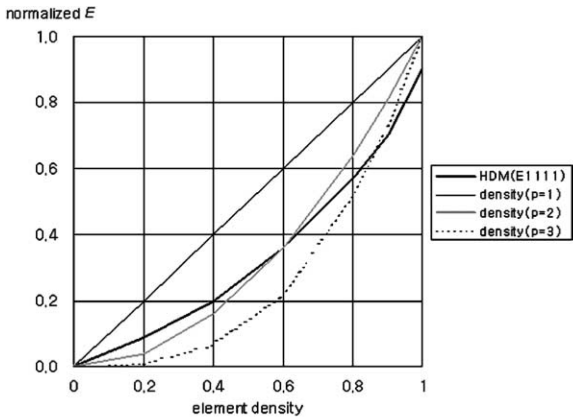

Fig. 3. Elastic moduli according to the element density.

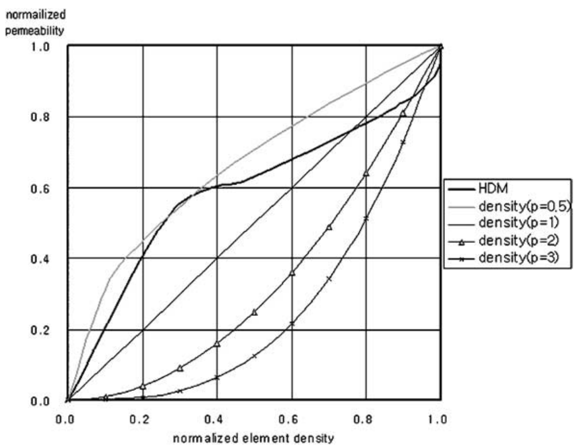

Fig. 4. Magnetic permeability according to the element density.

4. Optimization process

4.1. Formulation of the optimization problem

Having the magnetic permeability value either by HDM or by the modified density method, the optimization process can be succeeded. The design objective is dependent on the problem interested and usually formulated based on the energy in the design domain. The specified volume ratio of the design domain is used to determine the volume constraint.

If the saturation effect of the material is considered, the value of magnetic permeability becomes a function of magnetic flux density. Therefore, the magnetic energy in region V can be defined as

$$W_m = \frac{1}{2}\int_V \int_0^{\boldsymbol{B}} \frac{1}{\mu(\boldsymbol{B})}\boldsymbol{B}\,\mathrm{d}\boldsymbol{B}\,\mathrm{d}v = \frac{1}{2}\int_V \boldsymbol{B}^{\mathrm{T}}\frac{1}{\mu(\boldsymbol{B})}\boldsymbol{B}\,\mathrm{d}v \tag{11}$$

where $\mu(\boldsymbol{B})$ is the magnetic permeability considering the saturation effect. It is computed either by Eq. (4) or by Eq. (10) according to the method for the optimal design. Setting L and I be inductance and current, respectively, the magnetic energy may be expressed as (see, Yoo et al., 2001);

$$W_m = \frac{1}{2}\int_V \boldsymbol{B}^{\mathrm{T}}\frac{1}{\mu(\boldsymbol{B})}\boldsymbol{B}\,\mathrm{d}v = \frac{1}{2}LI^2 \tag{12}$$

Using the relation between the inductance and the magnetic flux density, it can be also expressed as

$$W_m = \tfrac{1}{2}NI\psi \tag{13}$$

where N is the number of turns and ψ represents the magnetic flux.

The objective function for topology optimization is defined as maximizing the magnetic mean compliance l_{MMC} that is same as maximizing the magnetic energy in the design domain satisfying the volume constraint. Therefore, the optimization problem is defined as follows:

$$\begin{aligned} &\underset{\boldsymbol{D}}{\text{maximize}}\ \ l_{\mathrm{MMC}} = NI\psi \\ &\text{subject to}\quad \sum_{e=1}^{N} v_e \geqslant V_0 \end{aligned} \tag{14}$$

where $\boldsymbol{D}$ stands for the design variable matrix and V_0 is the specified volume ratio. Maximizing the magnetic mean compliance is the same as maximizing magnetic flux because N and I are constant during the optimization process. Since the area where the magnetic flux flows is fixed in the structural optimization in magnetic fields, we can maximize the magnetic flux density $\boldsymbol{B}$ in the design domain by performing the optimization problem.

4.2. Optimization algorithm and sensitivity analysis

During the iteration for optimization, the values of design variables are continuously changed according to the values of objective function and design constraints. To determine the design variables for the next iteration, it is required to compute the sensitivity of the design variables in contrast to objective function and constraints.

The computation of the sensitivity is determined based on the total potential energy T_m expressed as

$$T_m = \frac{1}{2}\int_V \boldsymbol{B}^{\mathrm{T}}\frac{1}{\mu(\boldsymbol{B})}\boldsymbol{B}\,\mathrm{d}v - NI\psi = -\frac{1}{2}l_{\mathrm{MMC}} \tag{15}$$

From Eqs. (14) and (15), we can see that the maximizing the magnetic mean compliance is the same as minimizing the total potential energy. The sensitivity of the design variable $\boldsymbol{D}$ according to the total potential energy may be expressed as

$$\frac{\partial T_{\mathrm{m}}}{\partial \boldsymbol{D}} = \int_V \boldsymbol{B}^{\mathrm{T}} \frac{1}{\mu(\boldsymbol{B})} \frac{\partial \boldsymbol{B}}{\partial \boldsymbol{D}} \mathrm{d}v + \frac{1}{2} \int_V \boldsymbol{B}^{\mathrm{T}} \frac{\partial}{\partial \boldsymbol{D}} \left(\frac{1}{\mu(\boldsymbol{B})} \right) \boldsymbol{B} \,\mathrm{d}v - NI \frac{\partial \psi}{\partial \boldsymbol{D}} \tag{16}$$

The first and third term of Eq. (16) can be cancelled out considering Eqs. (12) and (13). If the saturation effect of the material is reflected, Eq. (16) can be rewritten as follows:

$$\frac{\partial T_{\mathrm{m}}}{\partial \boldsymbol{D}} = \frac{1}{2} \int_V \boldsymbol{B}^{\mathrm{T}} \frac{\partial}{\partial \boldsymbol{D}} \left(\frac{1}{\mu(\boldsymbol{B})} \right) \boldsymbol{B} \,\mathrm{d}v = \frac{1}{2} \int_V \boldsymbol{B}^{\mathrm{T}} \left[\frac{\partial}{\partial \boldsymbol{D}} \frac{1}{\mu(\boldsymbol{B})} + \frac{\partial}{\partial \boldsymbol{B}} \left(\frac{1}{\mu(\boldsymbol{B})} \right) \frac{\partial \boldsymbol{B}}{\partial \boldsymbol{D}} \right] \boldsymbol{B} \,\mathrm{d}v \tag{17}$$

In the modified density method, the design variables are d_1, d_2, d_3 which describes the hole in an element. Referring the expression for the magnetic flux density in Eq. (10), Eq. (17) can be expressed as follows:

$$\begin{aligned} \frac{\partial T_{\mathrm{m}}}{\partial d_i} &= \frac{1}{2} \int_V \boldsymbol{B}^{\mathrm{T}} \left[\frac{\partial}{\partial d_i} \frac{1}{\mu(\boldsymbol{B})} + \frac{\partial}{\partial \boldsymbol{B}} \left(\frac{1}{\mu(\boldsymbol{B})} \right) \frac{\partial \boldsymbol{B}}{\partial d_i} \right] \boldsymbol{B} \,\mathrm{d}v \\ &= \frac{1}{2} \int_V \boldsymbol{B}^{\mathrm{T}} \left[-\frac{\partial \mu(\boldsymbol{B})/\partial d_i}{\mu^2(\boldsymbol{B})} + \frac{\partial}{\partial \boldsymbol{B}} \left(\frac{1}{\mu(\boldsymbol{B})} \right) \frac{\partial \boldsymbol{B}}{\partial d_i} \right] \boldsymbol{B} \,\mathrm{d}v \\ &= \frac{1}{2} \int_V \boldsymbol{B}^{\mathrm{T}} \left[-\frac{\mu_0(\mu_{\mathrm{r}} - 1)}{\mu^2} p(1 - d_i d_j d_k)^{p-1}(-d_j d_k) + \frac{\partial}{\partial \boldsymbol{B}} \left(\frac{1}{\mu(\boldsymbol{B})} \right) \frac{\partial \boldsymbol{B}}{\partial d_i} \right] \boldsymbol{B} \,\mathrm{d}v, \\ & i,j,k = 1,2,3 \quad \text{and} \quad i \neq j \neq k \end{aligned} \tag{18}$$

Utilizing Eq. (15), the sensitivity of the design variables on the objective function can be obtained.

$$\frac{\partial l_{\mathrm{MMC}}}{\partial \boldsymbol{D}} = -2 \frac{\partial T_{\mathrm{m}}}{\partial \boldsymbol{D}} \tag{19}$$

In the structural optimization in magnetic fields, it is necessary to deal with large number of design variables. Since the magnetic density is determined as the curl of the magnetic vector potential, the sensitivity value can be negative. In this study, sequential linear programming (SLP) is used to satisfy these constraints.

SLP requires the move limit to approximate the feasible design domain. Thomas et al. (1992) set move limit as

$$\Delta \boldsymbol{D} = \mathrm{Max}(\varsigma \boldsymbol{D}, \Delta \boldsymbol{D}_{\min}) \tag{20}$$

where $\Delta \boldsymbol{D}$ is the move limit, $\Delta \boldsymbol{D}_{\min}$ is the minimum move limit, and ζ is a constant move ratio. In this work, ζ is initially set to 0.1 and $\Delta \boldsymbol{D}_{\min}$ is set to 10% of the maximum value of the design variable. Comparing to the optimality criteria method (OCM), the convergence rate of SLP is slower. However, by using SLP, the convergence of the optimization problem is guaranteed and the negative sensitivity value can be treated while OCM cannot treat the negative sensitivity value.

5. Numerical applications

In the previous sections, the formulation for the modified density approach is given and the effect of the penalization parameter is shortly examined. In this section, numerical applications of the modified density method as well as HDM are discussed.

Two different structures used in the previous works (Yoo and Kikuchi, 2000; Yoo et al., 2000, 2001) are studied for comparison: H-shaped iron magnet (hereafter, referred to as H-magnet) and C-shaped iron-core (hereafter, referred to as C-core). In the H-magnet case, the effect of the design domain together with the effect of the variation of the penalization parameter in the modified density approach is focused. In the C-core case, the variation of the results according to the discretization density is also studied. Finally, prospective application of the modified density approach is discussed.

5.1. H-magnet application

Fig. 5(a) shows the three-dimensional view of the H-magnet considered. Marrocco and Pironneau (1978) suggested an optimal shape design of the tip-part in such a way that the magnet produces a constant magnetic flux density for the sir-gap region. Yoo et al. (2000, 2001) treated the same problem to obtain the optimal topology of the tip-part maximizing magnetic energy in order to maximize the magnetic flux density. Due to the symmetry, the analysis and design are restricted to one-quarter of the total magnet as shown in Fig. 5(b). Since the large current density of 5×10^6 A/m^2 is applied at the copper coil, the saturation effect of the iron-core must be considered. The Dirchlet boundary condition is applied along Γ_0 and the Neumann boundary condition is applied along Γ_1. The finite element model is composed of three-layer hexahedral elements along z-direction.

Fig. 6 displays the design domain for the optimization. Two different design domains, design domain I and design domain II are suggested in order to compare the optimization results according to the variation of the design domain. The design domain I is restricted to the tip-part as same as the previous works (see,

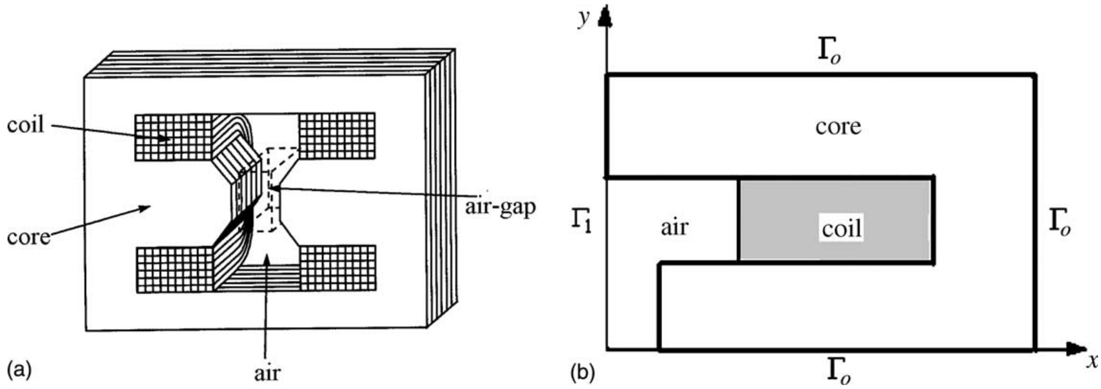

Fig. 5. Schematic view of an H-magnet for (a) three-dimensional cross-sectional view and (b) quarter model for analysis and design.

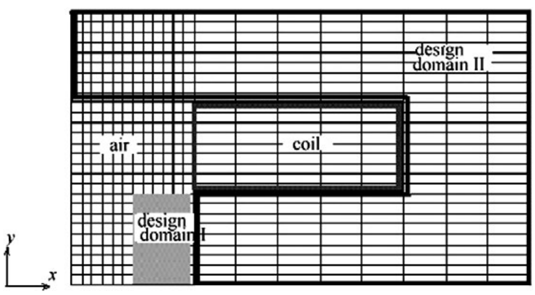

Fig. 6. Definition of the design domains for topology optimization.

e.g., Yoo et al., 2000) while the design domain II is defined as the whole iron-core including design domain I. The volume constraints for topology optimization are pre-determined as 60% of the design domain in both cases.

Fig. 7 shows the topology optimization results for design domain I. It displays the gray scale results for the first layer among the three layers of the finite element model of the H-magnet. The result by HDM is similar to the result by the modified density method with penalization parameter of 0.5 or 1. The optimal shape by the modified density approach with large values of penalization parameter such as 2 or 3 is different to the result by HDM. Fig. 8 shows the results of topology optimization for design domain II, the whole iron-core. It eliminates the elements whose final density is lower than 50% of the original density. As similar to the results for design domain I, the result by the modified density approach with penalization parameter of 1 or 0.5 is mostly matched with the result by HDM especially for the tip region even though there is some differences in other regions.

The comparison of CPU time and the number of iterations until convergence vs. the different optimization methods are displayed in Fig. 9. Both the iteration number and CPU time increase rapidly with large value of the penalization parameter for design domain II. If the results shown in Fig. 8 are applied, in the air-gap portion the magnetic flux density that is intended to maximize by topology optimization can be increased more by HDM or by the modified density approach with small value of the penalization parameter as shown in Table 1.

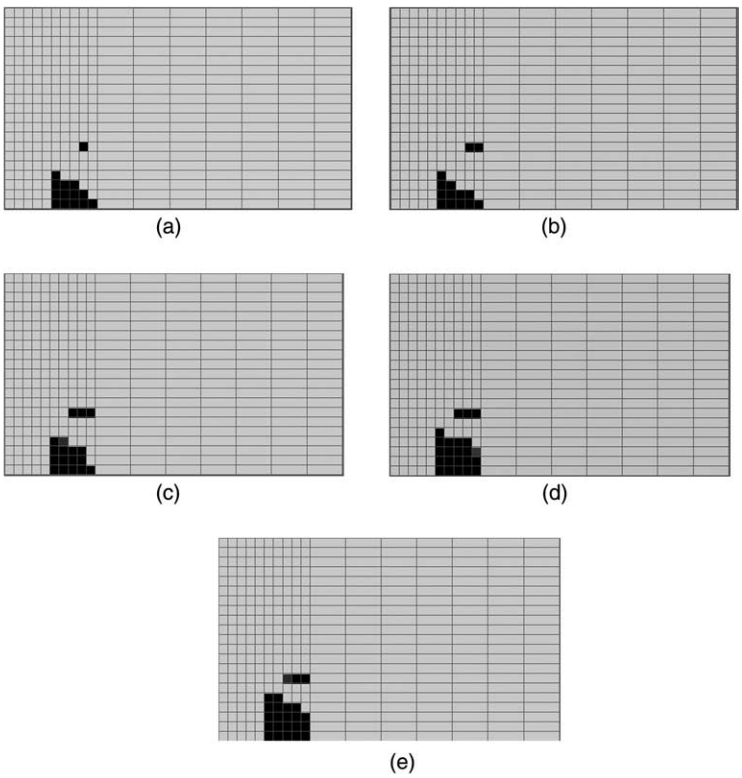

Fig. 7. Topology optimization results for design domain I by (a) HDM, (b) modified density method ($p = 0.5$), (c) modified density method ($p = 1$), (d) modified density method ($p = 2$) and (e) modified density method ($p = 3$).

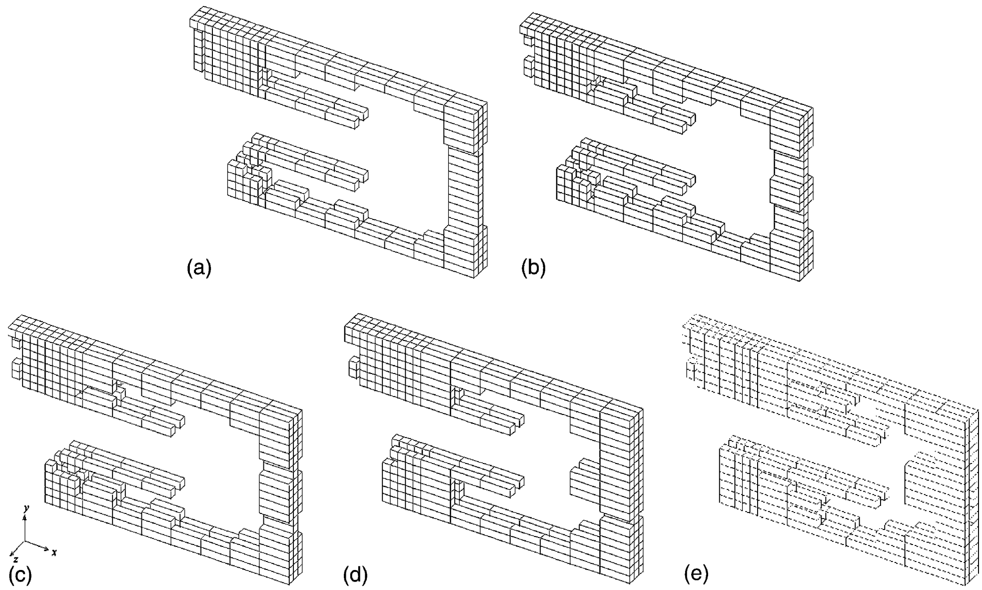

Fig. 8. Topology optimization results for design domain II by (a) HDM, (b) modified density method ($p = 0.5$), (c) modified density method ($p = 1$), (d) modified density method ($p = 2$) and (e) modified density method ($p = 3$).

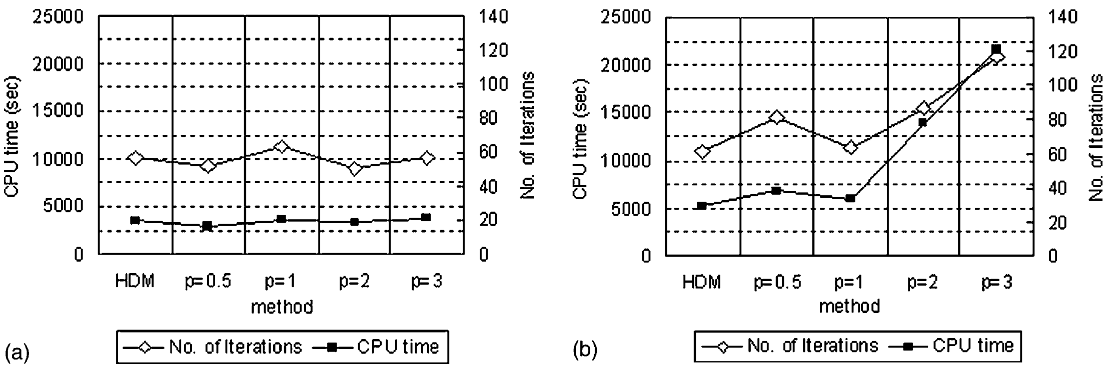

Fig. 9. Comparisons of CPU time and the number of iterations of the optimization result for (a) design domain I and (b) design domain II.

Table 1
Comparisons of flux density in the air-gap by HDM and the modified density approach (Wb/m^2)

		Averaged $\boldsymbol{B}_x$	Averaged $\boldsymbol{B}_y$
HDM		−1.221E−01	8.246E−02
Modified density method	$p = 0.5$	−1.221E−01	8.246E−02
	$p = 1$	−1.221E−01	8.246E−02
	$p = 2$	−1.209E−01	8.161E−02
	$p = 3$	−9.736E−02	6.573E−02

The particular tendency of the result can be predictable considering the graph in Fig. 4. The results by HDM and the modified density method with small penalization parameter such as 0.5 or 1 are superior to the results with large penalization parameter not only in CPU time and convergence rate but also in the final performance.

5.2. C-core application

The second example is a C-core excited by the current density applied to the wire coil around the air-gap as shown in Fig. 10(a). Salon and Istfan (1986) obtained the specified flux density by changing the width of the air-gap and the size of the wire. The design domain is defined at the whole iron-core part and the maximizing magnetic mean compliance in the design domain is obtained by changing the topology of the iron-core part. The model is discretized by one-layered 768 hexahedral elements as in Fig. 10(b). For the comparison, the model is also discretized by 456 elements to check the mesh-dependency.

Figs. 11 and 12 show the optimal shapes for the 456-element model and the 768-element model, respectively. Regarding the result by HDM for the 456-element model, it is similar to the result shown in the result by Yoo et al. (2000) for the tip-part adjacent to the air-gap. It is also similar to the result by HDM for the 768-element model since the result by HDM is not dependent on the mesh size. As shown in the figures, the results by HDM are similar to the results by modified density method with penalization parameter 0.5 or 1 even though there is some discrepancy. However, the results with the penalization parameter value as 2 or 3 are quite different to the results by HDM, especially at the tip-part.

The tendency shown in those figures can be verified by the comparison of CPU time and the number of iterations of each of the results as displayed in Fig. 13. As similar to the results in the previous section, we can observe the steep increase both at the number of iterations and at CPU time in the optimization results with 2 and 3 penalization parameter. CPU time of the 768-element model is almost 10 times larger than that of the 456-element model mostly due to the non-linear properties. The iteration numbers for the 768-element model with large penalization parameters such as 2 or 3 reach the upper limit designated as 150. Therefore, the results with those parameters are vague as shown in Fig. 12(d) and (e). The unclear topology at the tip-part is also displayed in Fig. 11(d) and (e).

The averaged flux density for 768-element model in the air-gap by HDM as well as the modified density method is shown in Table 2. Considering the initial shape of the C-core, the flux density to y-direction is

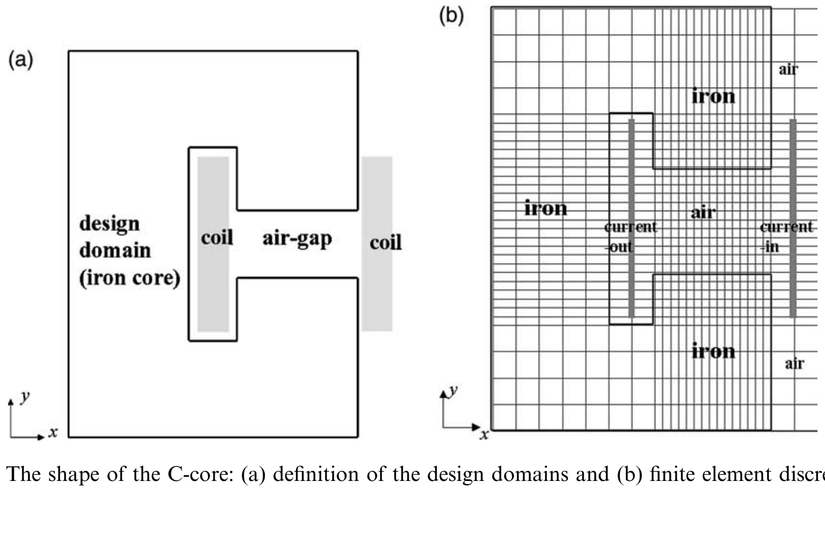

Fig. 10. The shape of the C-core: (a) definition of the design domains and (b) finite element discretization.

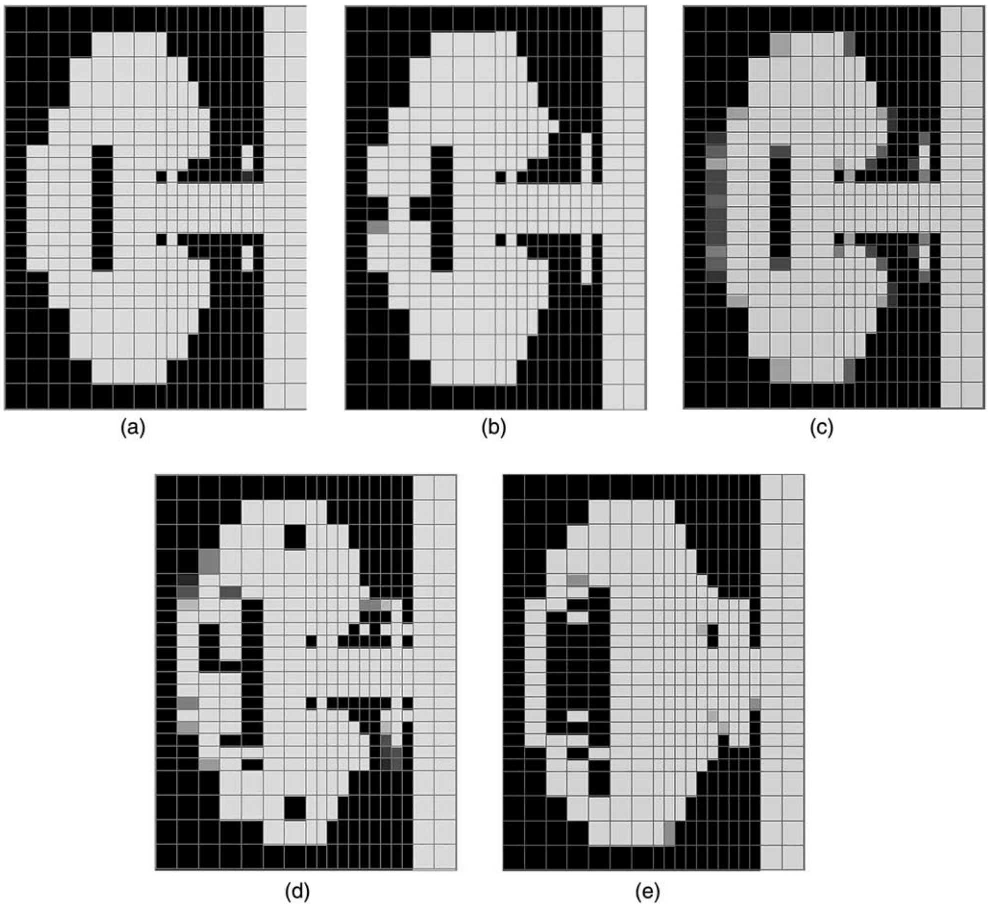

Fig. 11. Topology optimization results for 456-element model by (a) HDM, (b) modified density method ($p = 0.5$), (c) modified density method ($p = 1$), (d) modified density method ($p = 2$) and (e) modified density method ($p = 3$).

much larger than that to x-direction. As can be seen form the table, the average flux density especially average $\boldsymbol{B}_y$ is larger in case that HDM or the modified density method with 0.5 or 1 of penalization parameter is used due to the clear topology at the tip-part as confirmed in Figs. 11 and 12.

5.3. Discussion and suggestion

As confirmed in the previous sections, the modified density approach shows good results with small penalization parameter different to the ordinary topology optimization results in elastic fields. It is predictable by the comparison of Young's modulus and the magnetic permeability values according to the variation of the element density as designated in Figs. 3 and 4.

In the ordinary topology optimization in elastic cases such as maximizing stiffness, the final topology shows the stiffest structure with the pre-defined volume ratio. The elastic strain in optimal topology becomes minimum since the external forces applied are fixed. In topology optimization in magnetic fields, the design objective is maximizing the magnetic energy as defined in Eq. (14) and it is the same as maximizing the magnetic flux. Considering the similarity between the elastic strain and magnetic flux density in FEM,

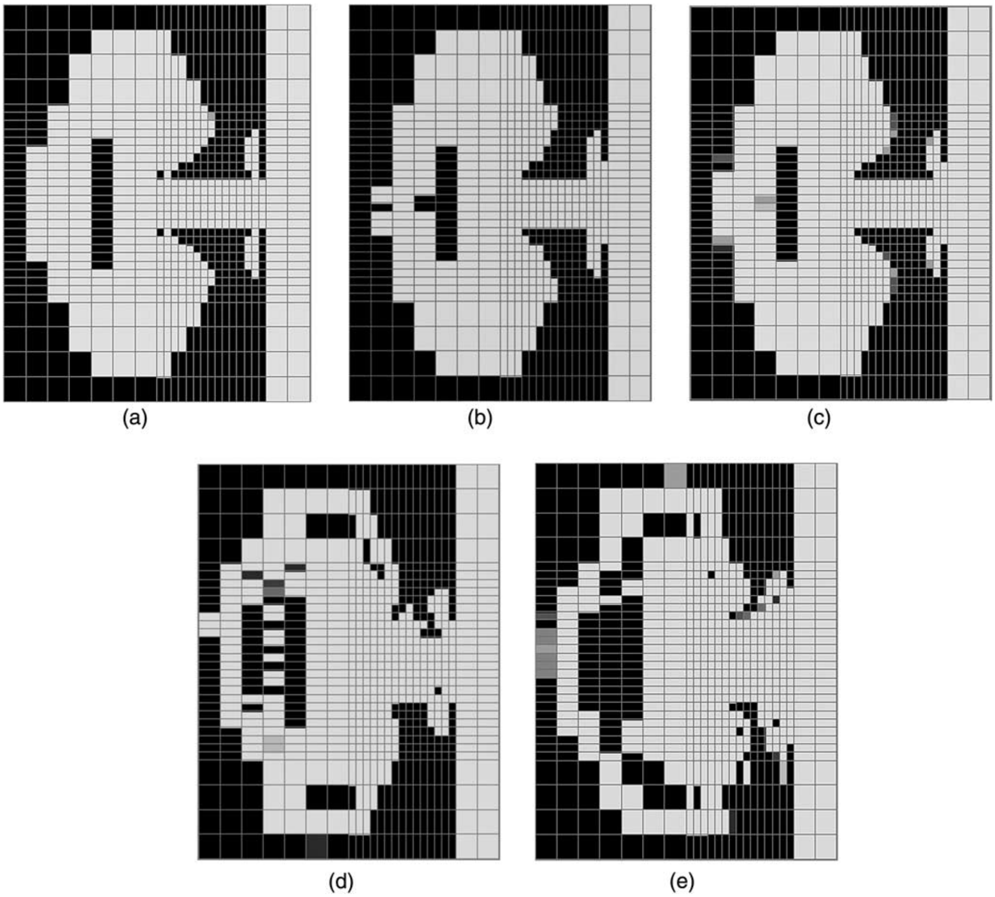

Fig. 12. Topology optimization results for 768-element model by (a) HDM, (b) modified density method ($p = 0.5$), (c) modified density method ($p = 1$), (d) modified density method ($p = 2$) and (e) modified density method ($p = 3$).

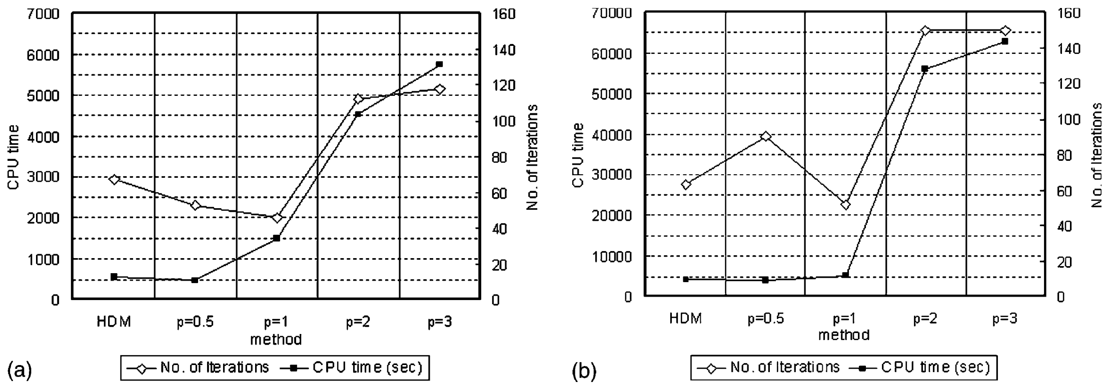

Fig. 13. Comparisons of CPU time and the number of iterations of the optimization result for (a) 456-element model and (b) 768-element model.

Table 2
Comparison between the results by HDM and the modified density method for 768-element model (Wb/m^2)

		Averaged $\boldsymbol{B}_x$	Averaged $\boldsymbol{B}_y$
HDM		1.015E−07	6.057E−02
Modified density method	$p = 0.5$	1.029E−07	6.057E−02
	$p = 1$	1.235E−07	6.063E−02
	$p = 2$	3.369E−04	5.129E−02
	$p = 3$	7.889E−05	5.707E−02

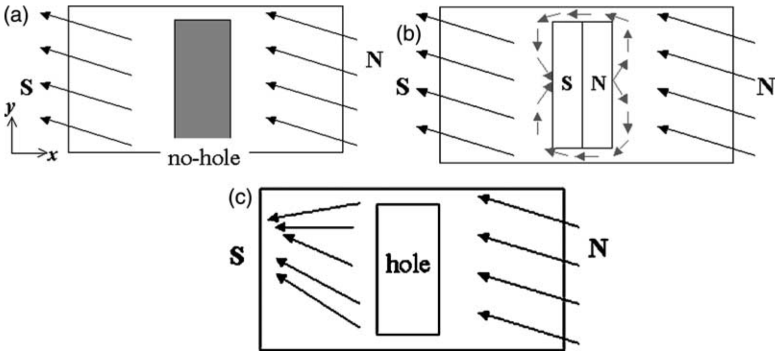

Fig. 14. The effect of hole in magnetic fields: (a) no hole, (b) equivalent magnetic effect and (c) final flux density.

we can attribute the difference in topology optimization to the difference of the design objective, maximizing and minimizing the energy in the design domain.

Another difference we should consider is the physical difference between elastic and magnetic fields. The magnetic field generated from external electric fields or permanent magnetization has a dipole shape pointing from the north to the south pole. Therefore, the magnetic flux is determined as curl of the magnetic vector potential. On the contrary, the elastic strain or the deformation does not have such characteristics and the elastic strain is determined as the divergence of the deformation. Further study is required to clarify the reason of the difference in the topology optimization result between elastic and magnetic fields. Nevertheless, it is necessary to consider the physical difference in magnetic fields if we try to apply the method generally used in elastic fields into magnetic fields.

Assuming a hole in the middle of material, the contour of elastic strain flows around the hole and it is the cause of the stress concentration. This tendency is similar to the contour of magnetic vector potential. However, if the width of the hole is small, the magnetic flux usually passes through the hole instead of passing around the hole and the direction of the magnetic flux can be changed. Fig. 14 shows the conceptual figure of the hole-effect. If there is no hole inside of a ferromagnetic material, the flux density keeps the same direction in the flow. If there is a hole in the material, the hole can be replaced with an equivalent magnet as shown in Fig. 14(b) and the direction of the original magnetic flux density can be changed into the direction of the vector sum. It is just a brief discussion of the effect of the hole in the ferromagnetic material: however, we can use this magnetic property in the structural design to control the direction of the magnetic flux density using the modified density method suggested. For example, at the beginning stage of the optimization process, we can expect better results by fixing some specific design variables designating the hole-size of an element to control the direction of the magnetic flux or the electromagnetic wave.

6. Conclusion

A modified density method, which can be used in the structural optimization in magnetic fields, is suggested in this study. It is possible to consider the orthotropic material property by applying the new density approach. We can expand the application of the modified density method into the electromagnetic field problems such as wave or the ordinary structural optimization in magnetic fields to control the direction of the magnetic flux.

For topology optimization using the density method, this work may be the first trial to clarify the difference between the magnetic field problem and the elastic field problem. The results varying the value of the penalization parameter are compared with the results by HDM. Different to the ordinary elastic field problems, we can confirm that the final results with small penalization parameter are superior to the results with large penalization parameter not only in the performance but also in the CPU time and the convergence rate. Even though a detail mathematical proof regarding the difference has not been studied, it is at least verified that it is necessary to consider the physical characteristics in magnetic fields if we hope to apply the density method to obtain the optimal shape of a structure in magnetic fields.

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